Comparison and development of efficient sampling methods for l1-minimization in the framework of polynomial chaos expansion

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A reliable approach to analyzing complex engineering systems requires understanding how various Quantities of Interest (QoI) depend upon system inputs that are often uncertain. Uncertainty Quantification is a field that aims at addressing these issues in a practical and rigorous manner, giving a meaningful characterization of uncertainties from the available information and admitting efficient propagation of these uncertainties for a quantitative validation of model predictions. In recent years, the polynomial chaos method (PC) became very popular and proved as a useful tool because it is capable of analyzing complex models in terms of parameter uncertainties far more efficient than traditional Monte Carlo methods. The further development of PC with regard to the treatment of high-dimensional and multistable systems with discontinuous transfer functions is subject of ongoing research. In general, PC is based on the construction of a polynomial surrogate of the models transfer function to be investigated. Due to the curse of dimensionality, the number of basic functions and thus also the PC coefficients increases strongly with the number of random parameters and with the complexity of the function to be approximated. The PC coefficients \([\mathbf{U}]\), can be determined by sampling the random space and solving the associated regression problem. In most cases, PC exhibits sparsity in that a small fraction of expansion coefficients are significant. By exploiting this sparsity compressive sampling provides a natural framework using relatively few evaluations of the QoI and in a manner that does not require intrusion into legacy solvers. Considering iterative PC algorithms, the PC or design matrix \([\mathbf{\Psi}]\) is constructed iteratively by successively adding basis functions. Each column represents a basis function and each row a sample in the random space. The right hand side \([\mathbf{Q}]\) of the regression problem are given by the (probably) expensive model evaluations of the system under investigation. Hence, the sampling scheme determines the efficacy and accuracy of the l1-minimizers to determine the PC coefficients. The goal of the thesis is to compare different sampling strategies in terms of their efficacy and stability. The schemes will be implemented into the existing python implementation pygpc and applied on different test problems. Finally, a new improved sampling scheme suitable for PC should be proposed and compared to existing ones from the literature.

The thesis will be conducted in cooperation with the MPI-CBS in Leipzig.

Tasks:
- Getting familiar with the Python toolboxes pygpc and fastmat
- Literature research on existing improved sampling schemes
- Implementation of selected sampling schemes in pygpc and fastmat
- Development of a new improved sampling scheme suitable for PC
- Application and comparison of the sampling schemes on various test problems

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